

Chemical Potential Dependence of Chiral Quark Condensate in Dyson-Schwinger Equation Approach of QCD

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Abstract

We propose a chemical potential dependent effective gluon propagator and study the chiral quark condensate in strongly interacting matter in the framework of Dyson-Schwinger equation approach. The obtained results manifest that, as the effect of the chemical potential on the effective gluon propagator is taken into account, the chiral quark condensate decreases gradually with the increasing of the chemical potential if it is less than the critical value, and the condensate vanishes suddenly at the critical chemical potential. The inclusion of the chemical potential in the effective gluon propagator enhances the decreasing rate and decreases the critical chemical potential. It indicates that the chiral symmetry can be restored

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completely at a critical chemical potential and restored partially as the chemical potential is less than the critical value. If the effective gluon propagator is independent of the chemical potential, the chiral symmetry can only be restored suddenly but no gradual restoration.

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The chiral symmetry spontaneous breaking and color confinement have been known as two essential characters of strong interaction in low energy region. The relation between these two essential properties is a fundamental problem in strong interaction physics. However, it is still very difficult to study such a relation directly at present. On the other hand, it is fortunate that one has known the vacuum of strong interaction would become trivial at high enough temperature and/or chemical potential because of the very weak-coupling interaction, or in other word, the asymptotic freedom. Then the behaviors of the chiral symmetry restoration and color deconfinement at high temperature and/or chemical potential would give signatures or information about the relation between the two properties. Concerning the measurement of the chiral symmetry restoration, one used to take the chiral quark condensate $\langle \bar{q}q \rangle$ as a characteristic since it is believed to be one of the most important configuration of the strong interaction vacuum (see for example Refs.[1, 2]). As for the methodology, the Lattice QCD simulation can be safely expand to finite temperature[3, 4] and has made great efforts to deal with the problem at finite chemical potential[5, 6, 7, 8, 9, 10, 11]. With the difference between the fundamental and adjoint representations of the color symmetry of a quark being taken into account, lattice QCD simulation and SU(3) gauge theory analysis show that the quark deconfinement and chiral symmetry restoration can happen at same critical temperature or not[3, 12]. Meanwhile many approaches in the framework of continuous field theory of QCD have been implemented to study the chiral symmetry breaking and its restoration in dense strongly interacting matter at zero temperature, and different models gave different behaviors at finite chemical potential (see for example Refs.[13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]). However, there is still not any work to take into account the effect of the chemical potential dependence of the effective gluon propagator.

It has been shown that the Dyson-Schwinger (D-S) equation approach provides a non-perturbative framework which admits a simultaneous study of dynamical chiral symmetry breaking and color confinement[27, 28]. With nonperturbative truncations preserving the chiral symmetry, this approach has been widely used to study the properties of strong interaction vacuum and the properties of hadrons in free space[29, 30]. Meanwhile it has

been extended to the system with finite temperature and/or finite chemical potential to simulate the chiral symmetry restoration and deconfinement[31, 32]. In this paper we will then take the Dyson-Schwinger equation approach at finite chemical potential to calculate the chemical potential dependence of the chiral quark condensate in strongly interacting matter. In previous studies on the chiral behavior at finite chemical potential, the effective gluon propagator is usually taken as the same as that in free space. We should know that the chemical potential does infect the interaction. Then the chemical potential dependence of the gluon propagator has to be deliberated. In this paper we propose an approximation to take into account the chemical potential dependence of the effective gluon propagator and investigate the effect of the interaction strength and the screening parameters in the modified effective gluon propagator on the chiral quark condensate at finite chemical potential.

As the lowest dimension condensate of quarks and gluons, the chiral quark condensate is the essential characteristic in the QCD phase transition, and hence plays essential role in describing hadron structure and properties of nuclear matter and finite nuclei. In the chiral limit, the chiral quark condensate can be evaluated from the quark mass function in ultraviolet region at zero chemical potential or with the definition

$$|\langle \bar{q}q \rangle| = Tr_{D,C} \int \frac{d^4q}{(2\pi)^4} G(q), \quad (1)$$

where $G(q)$ is the dressed quark propagator at zero chemical potential. It has been shown that, with the renormalized D-S equation, such an expression is equivalent to the one deduced from the mass function in QCD Sum rules[33]. In the case of finite chemical potential, one usually takes the same expression to study the chemical potential dependence of the chiral quark condensate[20, 24, 33]. We have then

$$|\langle \bar{q}q \rangle_\mu| = Tr_{D,C} \int \frac{d^4q}{(2\pi)^4} \mathcal{G}[\mu](q), \quad (2)$$

where $\mathcal{G}[\mu](q)$ is the dressed quark propagator at finite chemical potential. It is then imperative to investigate the chemical potential dependence of the dressed quark propagator.

The main aim of D-S equation approach is just to study the propagators of fermion and boson at all energy scale. The D-S equation approach is based on a coupled set of integral equations among quark, gluon, ghost and vertex functions. They form a countably infinite set of coupled integral equations with the one for a n -point Schwinger function depending on the $(n + 1)$ and higher point functions. In order to handle the system practically, it is necessary to make certain simplifications and truncations. One simple approach, which is commonly referred to as the rainbow approximation, employs the bare quark-gluon vertex and leaves the equation for the dressed quark propagator to be solved with a given effective gluon propagator as input. Under the rainbow approximation, the dressed quark propagator at zero chemical potential $G(p) \equiv \mathcal{G}[\mu = 0](p)$ can be determined well with the truncated D-S equation[34, 35, 36]

$$G^{-1}(p) = i\gamma \cdot p + \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} D_{\mu\nu}(p-q) \gamma_\mu G(q) \gamma_\nu. \quad (3)$$

where $D_{\mu\nu}(k)$ is an effective gluon propagator, and the inverse of the dressed quark propagator is conventionally decomposed as

$$G^{-1}(p) = i\gamma \cdot p A(p^2) + B(p^2), \quad (4)$$

with $A(p^2)$ and $B(p^2)$ being scalar functions. The chiral quark condensate at zero chemical potential can be determined by the two scalar functions as:

$$|\langle \bar{q}q \rangle| = 12 \int \frac{d^4q}{(2\pi)^4} \frac{B(q^2)}{q^2 A^2(q^2) + B^2(q^2)}. \quad (5)$$

Basing on the solution of the D-S equation, one can obtain a trivial solution $B(p^2) = 0$, namely the Wigner solution that characterizes a phase in which chiral symmetry is not broken and the dressed quarks are not confined. This phase relates to the trivial vacuum with $\langle \bar{q}q \rangle \equiv 0$. We are mainly interested in the nontrivial solution with nonzero B (Nambu solution) which should be found by numerical iterations. This solution corresponds to the nontrivial vacuum containing the dynamical quark mass generation, the chiral quark condensate and the existence of massless Goldstone bosons. It is also necessary to mention that the Wigner solution is always possible due to the path-integral formula used in the framework of the D-S equation. Nevertheless, the Nambu solution is only possible if

the coupling is strong enough at the infrared region[27, 37, 38]. Since the chiral quark condensate is zero in Wigner phase and nonzero in Nambu phase, the variation behavior of the chiral quark condensate with respect to the chemical potential can simulate the chiral phase transition from the Nambu phase to Wigner phase.

The chemical potential is introduced as a Lagrange multiplier $\mu\bar{q}\gamma_4q$ in QCD action. Similarly, the dressed quark propagator $\mathcal{G}[\mu](p)$ at non-zero chemical potential μ can be written as

$$\mathcal{G}^{-1}[\mu](p) = i\gamma \cdot p - \gamma_4\mu + \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} D_{\mu\nu}(p-q)\gamma_\mu\mathcal{G}[\mu](q)\gamma_\nu, \quad (6)$$

where the form of the effective interaction $D_{\mu\nu}$ is usually taken as the same as that in Eq. (3).

In convenient approach of D-S equation at finite chemical potential, the dressed-quark propagator can be written, in general, as[20, 31]

$$\mathcal{G}^{-1}[\mu](p) = i\vec{\gamma} \cdot \vec{p}A(\tilde{p}) + i\gamma_4(p_4 + i\mu)C(\tilde{p}) + B(\tilde{p}), \quad (7)$$

where $\tilde{p}_\nu = (\vec{p}, p_4 + i\mu)$, the complex functions A , C and B can be defined by the quark equation with an effective gluon propagator. With a δ -function in momentum space for the effective gluon propagator (Munczek-Nemirovsky model [39]), it has been found that, in the D-S equation approach [20, 24], the quark condensate increases with chemical potential, which is not consistent with the behavior given in composite operator approach of QCD [13], dilute instanton liquid model [21], Nambu–Jona-Lasinio (NJL) model [23] and other approaches [14, 16, 22, 25]. Looking over the characteristic of the Munczek-Nemirovsky model, which gives an infinity at zero exchanged momentum, we propose that the discrepancy of the variation behaviors of the chiral quark condensate against the chemical potential arises from the overestimated sudden enhancement in the far infrared region. To remove the discrepancy, we should then suppress the far infrared enhancement. Convenient works have shown that the model with effective gluon propagator

$$D_{\mu\nu}(k) = t_{\mu\nu}4\pi^2d\frac{\chi^2}{k^4 + \Delta}, \quad (8)$$

with $d = \frac{12}{27}$ and $t_{\mu\nu} = \delta_{\mu\nu}$, which means that the “Feynman-like” gauge is taken in practical calculation, can describe the pion weak decay constant and other low energy chiral

observables well (see for example Ref.[40]). It is obvious that this effective gluon propagator has an finite infrared enhancement and does not involve the ultraviolet behavior of the QCD running coupling. Since such a model produces ultraviolet convergent integrals naturally, the renormalization is not necessary. Meanwhile, this effective gluon propagator does not have a Lehmann representation, and the corresponding classical potential between quarks can be written as

$$V(r) = -\frac{d\pi\chi^2}{r\sqrt{\Delta}}e^{-r\sqrt[4]{\frac{\Delta}{4}}}\sin\left[r\sqrt[4]{\frac{\Delta}{4}}\right]. \quad (9)$$

It is apparent that such a potential corresponds to an approximately quadratic confinement in intermediate range (up to $r \approx 3.0$ fm) and linear-like in very short range ($r \lesssim 0.2$ fm). In most recent years, great progress has been made in systematic studies on quark-quark interaction (see for example Ref. [34, 35, 41, 42, 43, 44, 45]). One should, in principle, take the obtained analytic properties of the gluon and quark propagators (c.f. given in Ref. [34, 35]) to study the chiral phase transition. However, for simplicity in studying the quark D-S equation and incorporating the chemical potential effect at present stage, we take the effective gluon propagator in Eq. (8) as the starting point of our model of interaction.

In general approach to study the quark equation at finite chemical potential, one should take a chemical potential μ dependent effective gluon propagator as input, since the chemical potential of the matter also influences the interaction. However, the related knowledge is still in lack up to now. For simplicity, we propose an approximation for the effective gluon propagator at finite chemical potential with an extension $k^2 \rightarrow k^2 + \beta\mu^2$, the Eq. (8) is then modified as

$$D'_{\mu\nu}(k) = t_{\mu\nu}4\pi^2d\frac{\chi^2}{(k^2 + \beta\mu^2)^2 + \Delta}, \quad (10)$$

where μ denotes the chemical potential and β is a scaling parameter denoting the strength of the μ dependence. The classical potential with this modified gluon propagator can be written as

$$V'(r) = -\frac{d\pi\chi^2}{r\sqrt{\Delta}}e^{-r\sqrt{\frac{\sqrt{\beta^2\mu^4+\Delta}+\beta\mu^2}{2}}}\sin\left[r\sqrt{\frac{\sqrt{\beta^2\mu^4+\Delta}-\beta\mu^2}{2}}\right]. \quad (11)$$

It is evident that the strength of the attractive interaction becomes smaller with an increase of the chemical potential and the scaling parameter β .

Substituting Eq. (10) into Eq. (6) and accomplishing some derivations, we have the equations for the scalar functions $A(\tilde{p})$, $B(\tilde{p})$ and $C(\tilde{p})$ in Eq. (7) as

$$(A(\tilde{p}) - 1)\tilde{p}^2 = \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} \frac{4\pi^2\chi^2 d}{(k^2 + \beta\mu^2)^2 + \Delta} \frac{A(\tilde{q})\vec{p} \cdot \vec{q}}{\tilde{q}^2 A^2(\tilde{q}) + (q_4 + i\mu)^2 C^2(\tilde{q}) + B^2(\tilde{q})}, \quad (12)$$

$$(C(\tilde{p}) - 1)(p_4 + i\mu) = \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} \frac{4\pi^2\chi^2 d}{(k^2 + \beta\mu^2)^2 + \Delta} \frac{C(\tilde{q})(q_4 + i\mu)}{\tilde{q}^2 A^2(\tilde{q}) + (q_4 + i\mu)^2 C^2(\tilde{q}) + B^2(\tilde{q})}, \quad (13)$$

$$B(\tilde{p}) = \frac{16}{3} \int \frac{d^4q}{(2\pi)^4} \frac{4\pi^2\chi^2 d}{(k^2 + \beta\mu^2)^2 + \Delta} \frac{B(\tilde{q})}{\tilde{q}^2 A^2(\tilde{q}) + (q_4 + i\mu)^2 C^2(\tilde{q}) + B^2(\tilde{q})}. \quad (14)$$

With the parameter being taken as $\Delta = 0.01 \text{ GeV}^4$, $\chi = 1.33 \text{ GeV}$, with which the pion decay constant can be fitted to 87 MeV (quite close to the experimental date 93 MeV)[40], we can solve the coupled equations at zero chemical potential easily and obtain the chiral quark condensate in vacuum as $(250 \text{ MeV})^3$. It is evident that such a result agrees excellently well with the empirical value.

At nonzero chemical potential, it is a quite hard task to solve the coupled integral equations (12)-(14), since they depend on the momenta p , the chemical potential μ as well as the angle θ between p and μ with $\cos\theta = p \cdot \mu / \sqrt{p^2 \mu^2}$ due to the breaking of the $O(4)$ symmetry in the four momentum space. By abandoning the commonly used discretion of the unknown functions but implementing the *smooth polynomial approximations*[30, 46] for the to be determined complex functions $A(\tilde{p})$, $C(\tilde{p})$ and $B(\tilde{p})$, we solve the equations (12)-(14) (for the process of the smoothing and solving the equation, please see the Appendix-A).

With the solutions of the D-S equations at any chemical potential and scaling parameter β , we evaluate the chemical potential dependence of the chiral quark condensate at a series values of the scaling parameter β . A part of the obtained results are illustrated in Fig. 1. The figure shows apparently that, as the chemical potential increases in the region less than a critical value, the in-medium chiral quark condensate decreases monotonously

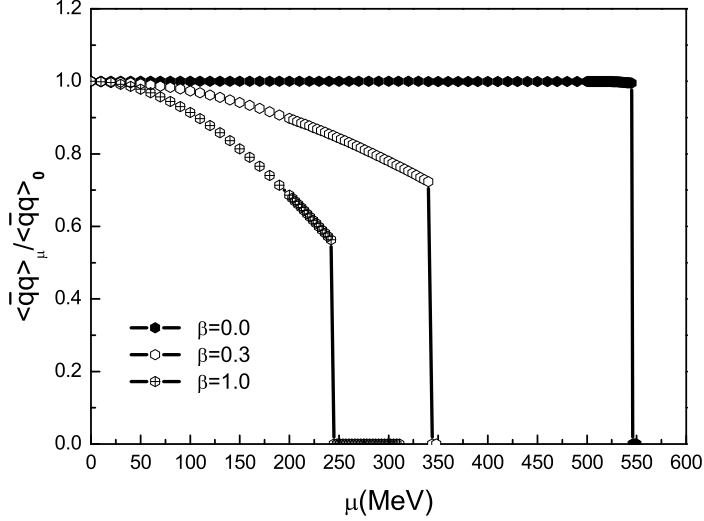


Figure 1: Calculated chemical potential dependence of the chiral quark condensate at several scaling parameters

for any value of $\beta \in (0, 1]$. As the critical value of the chemical potential μ_c is reached, the chiral quark condensate vanishes suddenly. Such a behavior indicates that the chiral symmetry can be restored partially before the critical chemical potential is reached. At the critical point of the chemical potential, the chiral symmetry can be completely restored suddenly. And the phase transition is in first order. Such a result is consistent with that given in the composite operator formalism of QCD [13]. Moreover, with the increase of the scaling parameter β , the decreasing rate of the chiral quark condensate against the chemical potential gets obviously larger and the critical chemical potential becomes apparently smaller. For instance, for $\beta = 0.3, 1.0$, the critical chemical potential μ_c is 344 MeV, 245 MeV and the ratio $|\langle \bar{q}q \rangle_{\mu_c}|/|\langle \bar{q}q \rangle_0|$ is 72.4%, 56.3%, respectively. In addition, in the case of that the chemical potential does not affect the effective gluon propagator, i.e., $\beta \equiv 0$, the chiral quark condensate in the matter maintains almost a constant if the chemical potential is less than the critical value ($\mu_c = 546$ MeV, $|\langle \bar{q}q \rangle_{\mu_c}|/|\langle \bar{q}q \rangle_0| = 99.6\%$). Such a result is analogous to that given in the NJL model [23] and dilute instanton liquid

model [21]. These results indicate that the critical chemical potential for the chiral quark condensate to be restored completely decreases with the increase of the scaling parameter β . The obtained result of the scaling parameter dependence of the critical chemical potential is illustrated in Fig. 2.

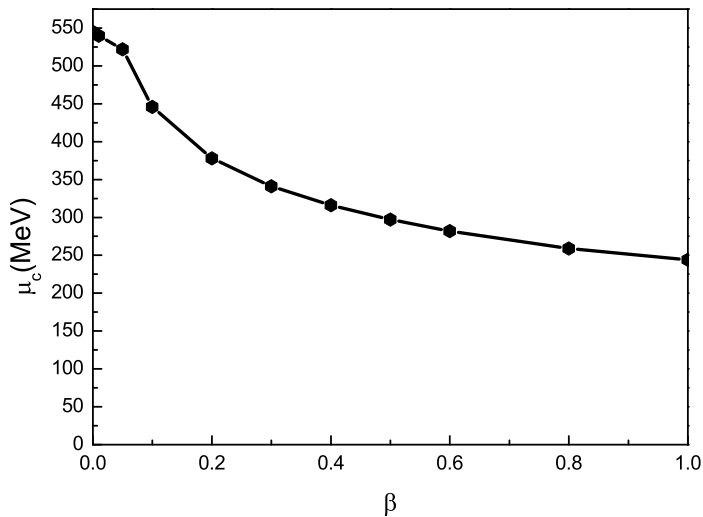


Figure 2: The scaling parameter dependence of the critical chemical potential for the chiral symmetry to be restored

To understand the variation characteristics of the chiral quark condensate against the chemical potential and the scaling parameter intuitively, we recall the definition of the condensate in Eq. (2). It is evident that the integrand for the condensate reads

$$\frac{p^2 B(\tilde{p})}{\tilde{p}^2 A^2(\tilde{p}) + (p_4 + i\mu)^2 C^2(\tilde{p}) + B^2(\tilde{p})}, \quad (15)$$

where the $A(\tilde{p})$, $C(\tilde{p})$ and $B(\tilde{p})$ are the solutions of the D-S equations (12)-(14). After integrating over the angle, one can express the momentum dependence of the integrand explicitly as (neglecting some constants)

$$IG(p^2) = \int_0^\pi d\theta \sin^2 \theta \frac{p^2 B(\tilde{p})}{\tilde{p}^2 A^2(\tilde{p}) + (p_4 + i\mu)^2 C^2(\tilde{p}) + B^2(\tilde{p})}, \quad (16)$$

with $\cos \theta = p \cdot \mu / \sqrt{p^2 \mu^2}$. Since the real part of both $A(\tilde{p})$ and $C(\tilde{p})$ is even and increasing function of the $\cos \theta$, the real part of $B(\tilde{p})$ is an even but decreasing function of the $\cos \theta$, and the imaginary part of $A(\tilde{p})$, $C(\tilde{p})$ and $B(\tilde{p})$ is odd function of $\cos \theta$, such an integrand is a real function of the momentum p . The behavior of the function $IG(p^2)$ at several values of the chemical potential and the scaling parameter can be displayed in Fig. 3. It is

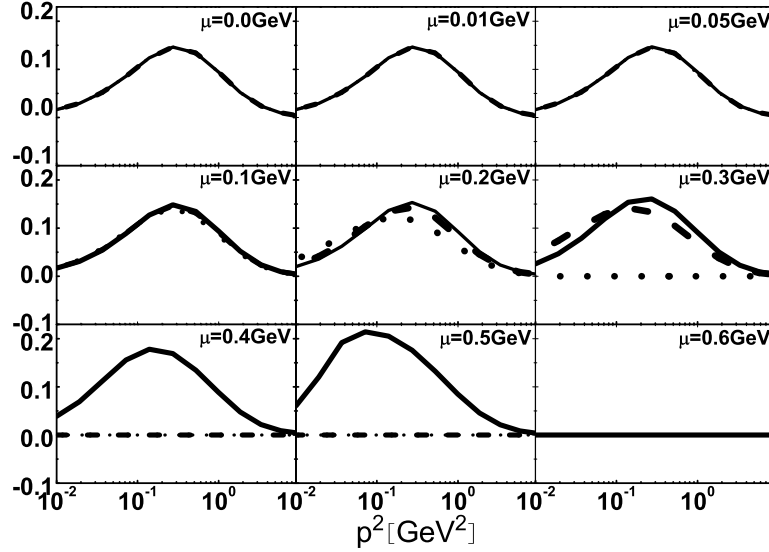


Figure 3: Calculated chemical potential dependence of $IG(p^2)$ at several scaling parameters: solid curve for $\beta = 0.0$; dash curve for $\beta = 0.3$; dot curve for $\beta = 1.0$.

evident that, at any chemical potential, the function $IG(p^2)$ is convergent in the infrared and the ultraviolet regions. It indicates that the condensate is well defined. In more detail, there exists a critical chemical potential μ_c , if $\mu > \mu_c$, $IG(p^2) \equiv 0$. Such a μ_c decreases with the increasing of the scaling parameter β , for example, $\mu_c = 0.546$ GeV for $\beta = 0.0$, $\mu_c = 0.344$ GeV for $\beta = 0.3$, $\mu_c = 0.245$ GeV for $\beta = 1.0$. In the case of $\beta \neq 0$, as $\mu < \mu_c$, $IG(p^2)$ increases with the increasing of momentum. However the $IG(p^2)$ decreases at large momentum. The momentum for the $IG(p^2)$ to take the maximal value shifts to smaller as the chemical potential and the scaling parameter increase. Moreover, the maximal value of $IG(p^2)$ decreases as the scaling parameter increases. These characteristics of

the integrand manifests evidently that, if the chemical potential is smaller than a critical value but influences the effective gluon propagator (i.e., with $\beta \in (0, 1]$), the chiral quark condensate decreases with the increasing of chemical potential, and the decreasing rate gets larger as the scaling parameter increases. As the chemical potential reaches the critical value, the chiral quark condensate vanishes suddenly. In the case of that the chemical potential does not influence the effective gluon propagator (i.e., with $\beta \equiv 0$), even though the solutions of the D-S equation at finite chemical potential are different from those at zero chemical potential, because of the compensation of the solutions A , C and B at the presently chosen parameters, the integrand of the chiral quark condensate behaves almost independent of the chemical potential if it is less than the critical value. The chiral quark condensate turns to be then approximately a constant before it vanishes suddenly at the critical chemical potential. On the other hand, concerning the mass function of a quark in the case of that the chemical potential does not affect the effective gluon propagator, we realize that it does not change drastically either with the chemical potential as it is smaller than the critical value. It means that the dressing effect come from the chiral quark condensate does not change obviously. It hints then the chiral quark condensate maintains almost a constant if the effective gluon propagator is independent of the chemical potential.

To explore the underlying physics of the variation behavior of the chiral quark condensate against the chemical potential and the scaling parameter, we look through the characteristic of the model effective gluon propagator. From Eqs. (10) and (11), one can recognize easily that the model effective gluon propagator is the one with approximately quadratic confinement in intermediate region and linear-like in very short region. Both the chemical potential and the scaling parameter play roles of screening on the confinement. It means that the increases of the chemical potential and the scaling parameter decrease the confinement strength. It has been known that the chiral quark condensate is proportional to the mean spectral density at zero energy of quarks with Banks-Casher relation[33, 47] $-\langle \bar{q}q \rangle = \pi \nu_q(0)$. In general principle, the mean spectral density $\nu(0)$ decreases as the attractive interaction gets weaker. The classical potential in Eq. (11) shows

that, in the case of that the effective gluon propagator depends on the chemical potential (i.e., with scaling parameter $\beta > 0$), the interaction in the system is just attractive and gets weaker as the chemical potential and the scaling parameter increase. Corresponding to the increases of the chemical potential and the scaling parameter, the mean spectral density at zero energy of quarks decreases, so that the absolute value of the chiral quark condensate decreases. Moreover, as the chemical potential is large enough (i.e., the density of quarks is sufficiently large), the screening effect is so strong that the interaction between quarks becomes very weak, even vanishes (asymptotic free), which is analogous to the molecular in a liquid, where it experiences an attraction to the inner if it is in the surface region and becomes almost free in the inner region. As a consequence, the chiral quark condensate changes to zero suddenly. Combining the calculated variation behavior of the chiral quark condensate with respect to the chemical potential (or the density of the strong interaction matter) and the inferred changing characteristic of the interaction between quarks, we can recognize that the chiral symmetry can be restored abruptly as the density of the matter reaches a critical value, and the chiral phase transition is in first order. In addition, the chiral symmetry can be restored partially and gradually before the phase transition takes place. However, in the case of that the effective gluon propagator is independent of the chemical potential (i.e., with $\beta \equiv 0$), the interaction is free from the chemical potential, or in other word, the chemical potential does not screen the attractive interaction. Then the chiral quark condensate almost maintains a constant before the critical chemical potential is reached.

In summary, we have calculated the chiral quark condensates in vacuum and in strongly interacting matter in the Dyson-Schwinger equation approach with a model effective gluon propagator including finite chemical potential effect in this paper. The calculated results show that, as the chemical potential influences the effective gluon propagator, the chiral quark condensate decreases with the increase of the chemical potential before the critical value is reached and the condensate vanishes suddenly at the critical chemical potential. Moreover, the increase of the scaling parameter of the chemical potential in the effective gluon propagator enhances the decreasing rate and decreases the

critical chemical potential. It indicates that the chiral symmetry can be restored abruptly as the density of the matter reaches a critical value, and the chiral phase transition is in first order. Before the critical density or chemical potential is reached, the chiral symmetry can be restored partially and gradually. In addition, as the chemical potential does not affect the effective gluon propagator, the chiral symmetry can only be restored suddenly at a critical chemical potential, but no gradual restoration takes place before the critical chemical potential is reached. It manifests that the effect of the chemical potential on the gluon propagator plays significant role in the process of the chiral symmetry restoration. Concerning the modification on the effective gluon propagator at finite chemical potential, we just extend the exchanged momentum related term from k^2 to chemical potential dependent $k^2 + \beta\mu^2$. The parameter β or the $\beta\mu^2$ displays the effect of the finite chemical potential on the effective gluon propagator or the quark-quark interaction. This is definitely only a model or an approximation. Furthermore, we have taken the rainbow approximation for the quark equation. Then studying the strong interaction matter by solving the coupled quark, gluon and ghost equations systematically is imperative. The related investigations are under progress.

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Appendix-A:

Because the $O(4)$ symmetry in the four momentum is broken, the complex functions $A(\tilde{p})$, $C(\tilde{p})$ and $B(\tilde{p})$ depend not only on the square of momentum p^2 and chemical potential μ^2 but also on the angle θ between p and μ with $\cos\theta = p \cdot \mu / \sqrt{p^2\mu^2}$. To solve the coupled equations (12)-(14), a smooth polynomial approximation should be taken for

both the angle and the square of momentum when we discrete the integrand in numerical calculation.

The angle θ dependence of the functions is analyzed by an expansion in terms of the second kind Chebyshev polynomials $U_i(\cos \theta)$ as

$$A(\tilde{p}^2) = \sum_{m=0}^{N_{ch,2}-1} A_m(p^2; \mu^2) U_m(\cos \theta) (i)^m, \quad (A.1)$$

$$C(\tilde{p}^2) = \sum_{m=0}^{N_{ch,2}-1} C_m(p^2; \mu^2) U_m(\cos \theta) (i)^m, \quad (A.2)$$

$$B(\tilde{p}^2) = \sum_{m=0}^{N_{ch,2}-1} B_m(p^2; \mu^2) U_m(\cos \theta) (i)^m. \quad (A.3)$$

Since the $U_k(\cos \theta)$ form an orthogonal set, we can project the equations (12), (13) and (14) onto a set of nonlinear integral equations satisfied by the form factors $A_m(p^2; \mu^2)$, $C_m(p^2; \mu^2)$ and $B_m(p^2; \mu^2)$ which are all *real* functions depending only on p^2 and μ^2 . In practical calculation we truncate the expansion at a certain rank $N_{ch,2}$ and the degree of the fitting can be mitigated by increasing $N_{ch,2}$. After the angular expansion we then take Chebyshev expansion for the form factors $A_m(p^2; \mu^2)$, $C_m(p^2; \mu^2)$ and $B_m(p^2; \mu^2)$, which gives

$$A_m(p^2; \mu^2) = \sum_{j=1}^{N_{ch,1}} a_m^j T_j(s(p)) - \frac{a_m^1}{2}, \quad (A.4)$$

$$C_m(p^2; \mu^2) = \sum_{j=1}^{N_{ch,1}} c_m^j T_j(s(p)) - \frac{c_m^1}{2}, \quad (A.5)$$

$$B_m(p^2; \mu^2) = \sum_{j=1}^{N_{ch,1}} b_m^j T_j(s(p)) - \frac{b_m^1}{2}, \quad (A.6)$$

with

$$s(p) = \frac{\log(p^2/\Lambda\epsilon)}{\log(\Lambda/\epsilon)},$$

where $T_j(s(p))$ is the j -rank Chebyshev polynomial of the first kind, Λ is the ultraviolet cutoff, and ϵ is the infrared cutoff. In this expansion we make use of the first kind Chebyshev polynomial with a cutoff $N_{ch,1}$. Up to now the unknown functions A , C and B have been expressed by the smooth polynomial with $3N_{ch,1}N_{ch,2}$ Chebyshev coefficients

a_m^j , c_m^j and b_m^j which would be numerically calculated by an effective iteration method. In practical calculation, we take $N_{ch,1} = 32$, $N_{ch,2} = 6$, $\Lambda = 10$ GeV and $\epsilon = 0.01$ GeV (considering the parameter $\Delta = 0.01\text{GeV}^4$ and $\epsilon^4 = 10^{-8}\text{GeV}^4$, such a choice of the cutoff is quite reasonable).

In order to determine the $3N_{ch,1}N_{ch,2}$ Chebyshev coefficients a_m^j , c_m^j and b_m^j , we require these integral equations to satisfy $N_{ch,1}$ fixed external momenta. To solve the nonlinear equations, we implement the Newton method. This method takes derivatives of the equations with respect to the unknowns to speed up the convergence. After some derivations, it is transformed to a set of $((3N_{ch,1}N_{ch,2}) \times (3N_{ch,1}N_{ch,2}))$ linear equations, which can be solved easily. As for the integration we introduce spherical coordinates, i.e. the volume element d^4q reads $2\pi q^3 dq \sin^2 \theta d\theta \sin \phi d\phi$. The integration ranges of the variables are: $q \in [\epsilon, \Lambda]$ and $\theta, \phi \in [0, \pi]$. The integration is calculated by the N-points Gauss-Legendre quadrature rule. The integrand may still not be smooth enough at the junctions of some intervals, it is then necessary to split the region for much higher accuracy. We split the integrand into three regions $[\epsilon^2, \min(p^2, q_f^2)]$, $[\min(p^2, q_f^2), \max(p^2, q_f^2)]$ and $[\max(p^2, q_f^2), \Lambda^2]$, where p^2 is the square of the external momentum and q_f^2 is defined by the possible zero of $\tilde{q}^2 A^2(\tilde{q}) + (q_4 + i\mu)^2 C^2(\tilde{q}) + B^2(\tilde{q})$ at $q_4 = 0$.

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